Op-Amp Sallen-Key Filter Design

Eric Danson

May 25, 2025

The relative sensitivity of y to x is defined as

$$S_x^y = \frac{\partial y/y}{\partial x/x} = \left(\frac{\partial y}{\partial x}\right)\frac{x}{y} \tag{1}$$

and is useful for optimizing component values in filter designs. Designing for low sensitivity is a way to minimize variations in filter parameters due to component tolerances.

1 Low-Pass Filter

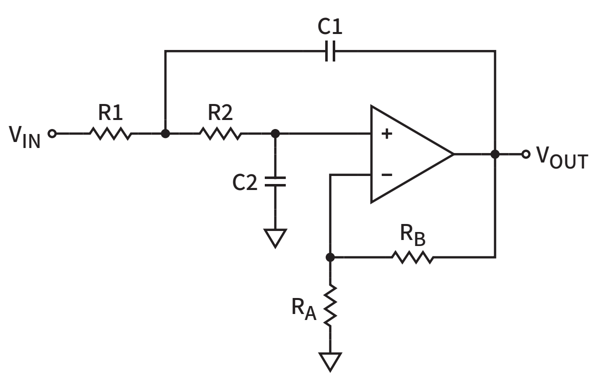

Fig. 1. Sallen-Key low-pass filter schematic.

$$H(s) = H_0 \frac{1}{s^2/{\omega_0}^2 + s/(\omega_0 Q) + 1} \tag{2}$$

$$H_0 = K = 1 + R_\mathrm{B}/R_\mathrm{A} \tag{3}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \tag{4}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(1-K) R_1 C_1 + (R_1 + R_2) C_2} \tag{5}$$

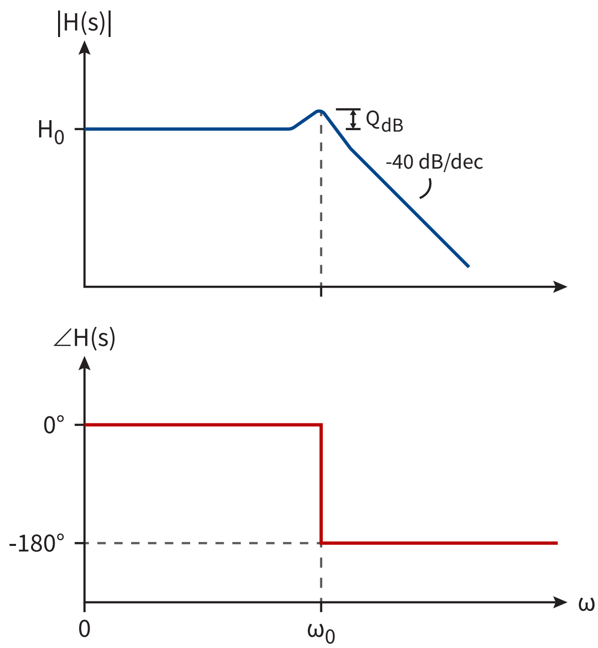

Fig. 2. Sallen-Key low-pass filter Bode plot.

1.1 SENSITIVITY

$$S^{\omega_0}_{R_1} = S^{\omega_0}_{R_2} = S^{\omega_0}_{C_1} = S^{\omega_0}_{C_2} = -\frac{1}{2} \tag{6}$$

$$S^{\omega_0}_{R_B} = S^{\omega_0}_{R_A} = 0 \tag{7}$$

$$S^{Q}_{R_1} = -S^{Q}_{R_2} = \omega_0 Q R_2 C_2 - \frac{1}{2} \tag{8}$$

$$S^{Q}_{C_1} = -S^{Q}_{C_2} = \omega_0 Q \left(R_1 C_2 + R_2 C_2\right) - \frac{1}{2} \tag{9}$$

$$S^{Q}_{R_B} = -S^{Q}_{R_A} = \omega_0 Q \left(K - 1\right) R_1 C_1 \tag{10}$$

1.2 COMPONENTS AS RATIOS

$$\begin{aligned} R_1 &= mR \\ R_2 &= R \\ C_1 &= nC \\ C_2 &= C \end{aligned} \tag{11}$$

$$\omega_0 = \frac{1}{RC\sqrt{mn}} \tag{12}$$

$$Q = \frac{\sqrt{mn}}{\left(1 - K\right) mn + m + 1} \tag{13}$$

1.2.1 *Minimizing Resistor Sensitivity*

$S^{Q}_{R_1} = S^{Q}_{R_2} = 0$ for

$$\begin{aligned} \omega_0 Q R_2 C_2 = \frac{1}{\left(1 - K\right) mn + m + 1} &= \frac{1}{2} \\ n &= \frac{m - 1}{\left(K - 1\right) m} \,. \end{aligned} \tag{14}$$

Substituting n into Q and solving for m,

$$m = 1 + 4Q^2 \left(K - 1\right) , \tag{15}$$

$$\implies n = \frac{4Q^2}{1 + 4Q^2 \left(K - 1\right)} \,. \tag{16}$$

If C is chosen, then R can be calculated from ω_0 as

$$R = \frac{1}{2\omega_0 QC}. \tag{17}$$

1.2.2 *Minimizing Capacitor Sensitivity*

$S^Q_{C_1} = S^Q_{C_2} = 0$ for

$$\omega_0 Q(R_1C_2 + R_2C_2) = \frac{m+1}{(1-K)\,mn + m + 1} = \frac{1}{2}. \tag{18}$$

However, the ratio expression has a minimum value of 1, so the minimum capacitor sensitivity is $S^Q_{C_1} = -S^Q_{C_2} = 1/2$ and occurs when $K = 1$.

1.2.3 *Monte Carlo Simulation*

The circuit used to generate the plot was designed for $K = 1$, $\omega_0 = 2\pi \cdot 1000$, and $Q = \sqrt{2}$ (Fig. 3). The resistor and capacitor tolerances were set to 1% and 5%, respectively.

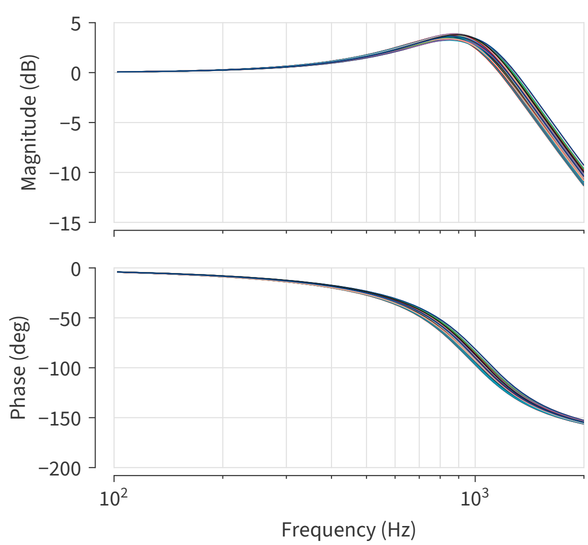

Fig. 3. Sallen-Key low-pass filter Bode plot of 100 Monte Carlo samples.

1.3 Resistors as ratios and equal capacitors

$$\begin{aligned} R_1 &= mR \\ R_2 &= R \\ C_1 = C_2 &= C \end{aligned} \tag{19}$$

$$\omega_0 = \frac{1}{RC\sqrt{m}} \tag{20}$$

$$Q = \frac{\sqrt{m}}{(2-K)\,m+1} \tag{21}$$

If C is chosen, then

$$m = \frac{4Q^2}{\left[1+\sqrt{1+4Q^2\,(K-2)}\right]^2}\,, \tag{22}$$

$$R = \frac{1+\sqrt{1+4Q^2\,(K-2)}}{2\omega_0 QC}\,. \tag{23}$$

1.4 Equal components

$$\begin{aligned} R_1 = R_2 &= R \\ C_1 = C_2 &= C \end{aligned} \tag{24}$$

$$\omega_0 = \frac{1}{RC} \tag{25}$$

$$Q = \frac{1}{3-K} \tag{26}$$

2 High-Pass Filter

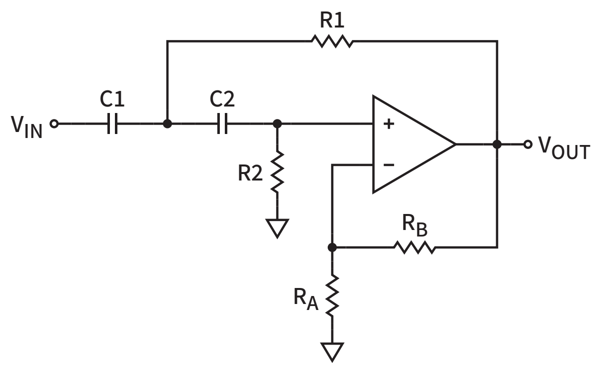

Fig. 4. Sallen-Key high-pass filter schematic.

$$H(s) = H_0 \frac{s^2/{\omega_0}^2}{s^2/{\omega_0}^2 + s/(\omega_0 Q) + 1} \tag{27}$$

$$H_0 = K = 1 + R_\mathrm{B}/R_\mathrm{A} \tag{28}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \tag{29}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(1-K)\, R_2 C_2 + R_1 (C_1 + C_2)} \tag{30}$$

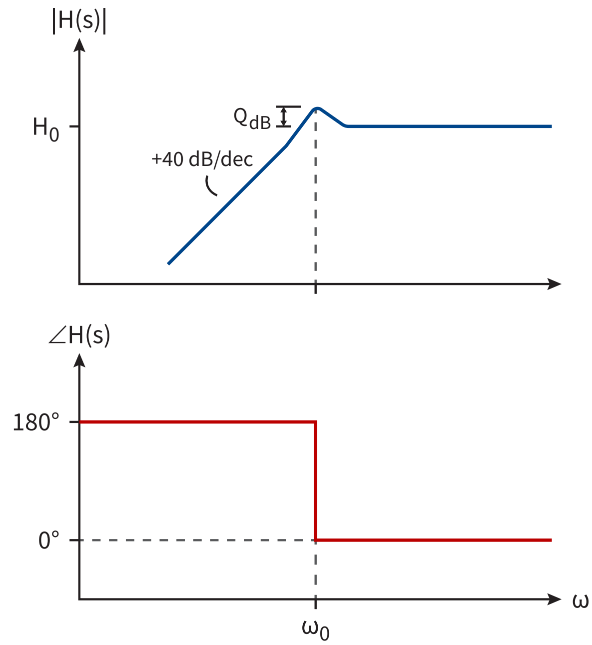

Fig. 5. Sallen-Key high-pass filter Bode plot.

2.1 SENSITIVITY

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \tag{31}$$

$$S_{R_B}^{\omega_0} = S_{R_A}^{\omega_0} = 0 \tag{32}$$

$$S_{R_2}^{Q} = -S_{R_1}^{Q} = \omega_0 Q (R_1 C_1 + R_1 C_2) - \frac{1}{2} \tag{33}$$

$$S_{C_2}^{Q} = -S_{C_1}^{Q} = \omega_0 Q R_1 C_1 - \frac{1}{2} \tag{34}$$

$$S_{R_B}^{Q} = -S_{R_A}^{Q} = \omega_0 Q (K-1) R_2 C_2 \tag{35}$$

2.2 COMPONENTS AS RATIOS

$$\begin{aligned} R_1 &= mR \\ R_2 &= R \\ C_1 &= nC \\ C_2 &= C \end{aligned} \tag{36}$$

$$\omega_0 = \frac{1}{RC\sqrt{mn}} \tag{37}$$

$$Q = \frac{\sqrt{mn}}{(1-K) + m(1+n)} \tag{38}$$

2.2.1 *Minimizing Capacitor Sensitivity*

$S^Q_{C_2} = S^Q_{C_1} = 0$ for

$$\begin{aligned} \omega_0 Q R_1 C_1 = \frac{mn}{(1-K) + m(1+n)} = \frac{1}{2} \\ n = \frac{(1-K) + m}{m} \,. \end{aligned} \tag{39}$$

Substituting n into Q and solving for m,

$$m = \frac{1}{4Q^2} + (K-1) \,, \tag{40}$$

$$\implies n = \frac{1}{1 + 4Q^2(K-1)} \,. \tag{41}$$

If C is chosen, then R can be calculated from ω_0 as

$$R = \frac{2Q}{\omega_0 C} \,. \tag{42}$$

2.2.2 *Minimizing Resistor Sensitivity*

$S^Q_{R_2} = S^Q_{R_1} = 0$ for

$$\omega_0 Q(R_1 C_1 + R_1 C_2) = \frac{m(1+n)}{(1-K) + m(1+n)} = \frac{1}{2} \,. \tag{43}$$

However, the ratio expression has a minimum value of 1, so the minimum resistor sensitivity is $S^Q_{R_2} = -S^Q_{R_1} = 1/2$ and occurs when $K = 1$.

2.2.3 *Monte Carlo Simulation*

The circuit used to generate the plot was designed for $K = 1$, $\omega_0 = 2\pi \cdot 1000$, and $Q = \sqrt{2}$ (Fig. 6). The resistor and capacitor tolerances were set to 1% and 5%, respectively.

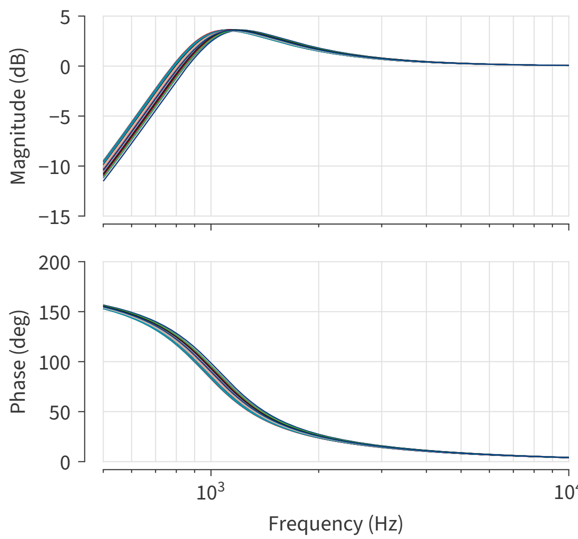

Fig. 6. Sallen-Key high-pass filter Bode plot of 100 Monte Carlo samples.

2.3 RESISTORS AS RATIOS AND EQUAL CAPACITORS

$$\begin{aligned} R_1 &= mR \\ R_2 &= R \\ C_1 = C_2 &= C \end{aligned} \tag{44}$$

$$\omega_0 = \frac{1}{RC\sqrt{m}} \tag{45}$$

$$Q = \frac{\sqrt{m}}{(1-K)+2m} \tag{46}$$

If C is chosen, then

$$m = \frac{\left[1+\sqrt{1+8Q^2\left(K-1\right)}\right]^2}{16Q^2}\,, \tag{47}$$

$$R = \frac{4Q}{\left[1+\sqrt{1+8Q^2\left(K-1\right)}\right]\omega_0 C}\,. \tag{48}$$

2.4 Equal Components

$$\begin{aligned} R_1 &= R_2 = R \\ C_1 &= C_2 = C \end{aligned} \tag{49}$$

$$\omega_0 = \frac{1}{RC} \tag{50}$$

$$Q = \frac{1}{3-K} \tag{51}$$

3 Band-Pass Filter

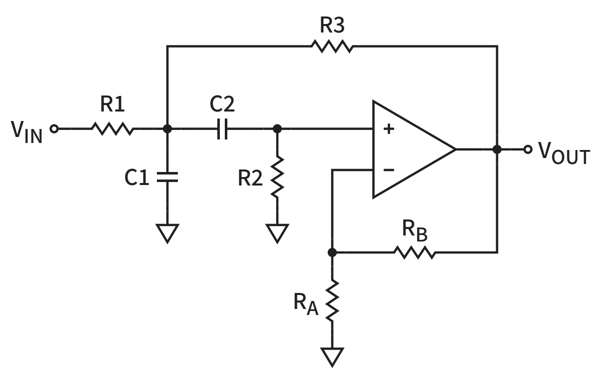

Fig. 7. Sallen-Key band-pass filter schematic.

$$H(s) = H_0 \frac{s/(\omega_0 Q)}{s^2/{\omega_0}^2 + s/(\omega_0 Q) + 1} \tag{52}$$

$$K = 1 + R_\mathrm{B}/R_\mathrm{A} \tag{53}$$

$$H_0 = \frac{K}{1 + (1-K) R_1/R_3 + (R_1/R_2)(1 + C_1/C_2)} \tag{54}$$

$$\omega_0 = \frac{\sqrt{1 + R_1/R_3}}{\sqrt{R_1 R_2 C_1 C_2}} \tag{55}$$

$$Q = \frac{\sqrt{1 + R_1/R_3} \cdot \sqrt{R_1 R_2 C_1 C_2}}{[1 + (1-K) R_1/R_3] R_2 C_2 + R_1 (C_1 + C_2)} \tag{56}$$

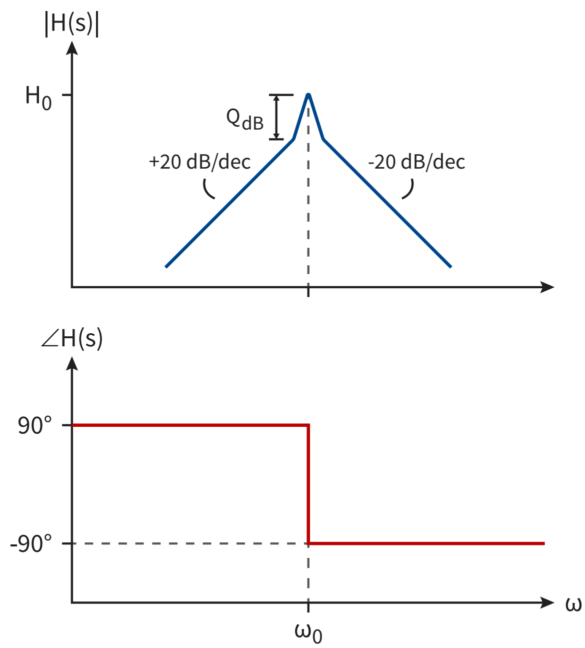

Fig. 8. Sallen-Key band-pass filter Bode plot.

3.1 SENSITIVITY

$$S_{R_1}^{H_0} = \frac{H_0}{K} - 1 \tag{57}$$

$$S_{R_2}^{H_0} = \frac{H_0}{K} \cdot \frac{R_1}{R_2} \left(1 + \frac{C_1}{C_2}\right) \tag{58}$$

$$S_{R_3}^{H_0} = \frac{H_0}{K} \cdot \frac{R_1}{R_3} (1 - K) \tag{59}$$

$$S_{C_2}^{H_0} = -S_{C_1}^{H_0} = \frac{H_0}{K} \cdot \frac{R_1 C_1}{R_2 C_2} \tag{60}$$

$$S_{R_B}^{H_0} = -S_{R_A}^{H_0} = \left(H_0 \frac{R_1}{R_3} + 1\right) \frac{K-1}{K} \tag{61}$$

$$S_{R_1}^{\omega_0} = -\frac{R_3}{2 (R_1 + R_3)} \tag{62}$$

$$S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \tag{63}$$

$$S_{R_3}^{\omega_0} = -\frac{R_1}{2 (R_1 + R_3)} \tag{64}$$

$$S_{R_B}^{\omega_0} = S_{R_A}^{\omega_0} = 0 \tag{65}$$

$$S_{R_1}^{Q} = S_{R_1}^{H_0} + S_{R_1}^{\omega_0} + 1 \tag{66}$$

$$S_{R_2}^{Q} = S_{R_2}^{H_0} + S_{R_2}^{\omega_0} \tag{67}$$

$$S_{R_3}^{Q} = S_{R_3}^{H_0} + S_{R_3}^{\omega_0} \tag{68}$$

$$S_{C_2}^{Q} = -S_{C_1}^{Q} = S_{C_2}^{H_0} + S_{C_2}^{\omega_0} \tag{69}$$

$$S_{R_B}^{Q} = -S_{R_A}^{Q} = S_{R_B}^{H_0} + S_{R_B}^{\omega_0} - \frac{K-1}{K} \tag{70}$$

3.2 COMPONENTS AS RATIOS

$$\begin{aligned} R_1 &= mR \\ R_2 &= nR \\ R_3 &= R \\ C_1 &= oC \\ C_2 &= C \end{aligned} \tag{71}$$

$$H_0 = \frac{K}{1 + (1-K)\,m + (1+o)\,m/n} \tag{72}$$

$$\omega_0 = \frac{\sqrt{1+m}}{RC\sqrt{mno}} \tag{73}$$

$$Q = \frac{\sqrt{1+m}\cdot\sqrt{mno}}{[1 + (1-K)\,m]\,n + m\,(1+o)} \tag{74}$$

3.2.1 *Minimizing Resistor and Capacitor Sensitivities*

$$S^Q_{R_1} = \frac{1}{1 + (1-K)\,m + (1+o)\,m/n} - \frac{1}{2\,(1+m)} \tag{75}$$

$$S^Q_{R_2} = \frac{(1+o)\,m/n}{1 + (1-K)\,m + (1+o)\,m/n} - \frac{1}{2} \tag{76}$$

$$S^Q_{R_3} = \frac{(1-K)\,m}{1 + (1-K)\,m + (1+o)\,m/n} - \frac{m}{2\,(1+m)} \tag{77}$$

$$S^Q_{C_2} = -S^Q_{C_1} = \frac{mo/n}{1 + (1-K)\,m + (1+o)\,m/n} - \frac{1}{2} \tag{78}$$

Minimizing the component sensitivities is more easily solved numerically with constraints set on the upper and lower bounds of the component ratios.

If C is chosen, then R can be calculated from ω_0 as

$$R = \frac{\sqrt{1+m}}{\omega_0 C\sqrt{mno}}\,. \tag{79}$$

3.2.2 *Example MATLAB Script*

```
%% Specifications

K = 1;
H0 = 0.5;
w0 = 2*pi*1e3;
Q = sqrt(2);
C = 10e-9;

%% Sensitivity optimization
```

```matlab

x0 = ones(1, 3); % Initial values for m, n, and o
lb = 0.01*ones(1, length(x0)); % Lower bound
ub = 100*ones(1, length(x0)); % Upper bound

fun_wrapper = @(x) fun(x, K);
nonlcon_wrapper = @(x) nonlcon(x, K, H0, Q);
[x, fval] = fmincon(fun_wrapper, x0, [], [], [], [], lb, ub, nonlcon_wrapper);

m = x(1);
n = x(2);
o = x(3);
R = sqrt(1 + m)/(w0*C*sqrt(m*n*o));

%% Functions

function f = fun(x, K)
    m = x(1);
    n = x(2);
    o = x(3);

    H0_K = 1/(1 + (1 - K)*m + (1 + o)*m/n);
    S_Q_R1 = H0_K - 1/(2*(1 + m));
    S_Q_R2 = H0_K*((1 + o)*m/n) - 1/2;
    S_Q_R3 = H0_K*((1 - K)*m) - m/(2*(1 + m));
    S_Q_C2 = H0_K*(m*o/n) - 1/2;
    S_Q_C1 = -S_Q_C2;

    % Sum of all the absolute sensitivities to be minimized
    f = abs(S_Q_R1) + abs(S_Q_R2) + abs(S_Q_R3) + abs(S_Q_C1) + abs(S_Q_C2);
end

function [c, ceq] = nonlcon(x, K, H0, Q)
    m = x(1);
    n = x(2);
    o = x(3);

    c = [];
```

```
    % Constrain the parameters to meet the specified H0 and Q
    ceq(1) = H0 - K/(1 + (1 - K)*m + (1 + o)*m/n);
    ceq(2) = Q - (sqrt(1 + m)*sqrt(m*n*o))/((1 + (1 - K)*m)*n
        + (1 + o)*m);
end
```

3.2.3 *Monte Carlo Simulation*

The circuit used to generate the plot was designed for $K = 1$, $H_0 = 0.5$, $\omega_0 = 2\pi \cdot 1000$, and $Q = \sqrt{2}$ (Fig. 9). The resistor and capacitor tolerances were set to 1% and 5%, respectively.

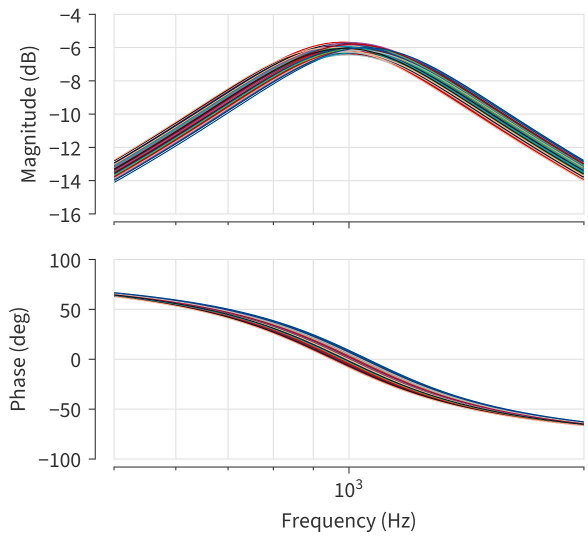

Fig. 9. Sallen-Key band-pass filter Bode plot of 100 Monte Carlo samples.

3.3 EQUAL CAPACITORS AND K = 1

$$\begin{aligned} C_1 = C_2 &= C \\ K &= 1 \end{aligned} \tag{80}$$

$$H_0 = \frac{1}{1 + 2R_1/R_2} \tag{81}$$

$$\omega_0 = \frac{\sqrt{1 + R_1/R_3}}{C\sqrt{R_1 R_2}} \tag{82}$$

$$Q = \frac{\sqrt{1 + R_1/R_3} \cdot \sqrt{R_1 R_2}}{2R_1 + R_2} \tag{83}$$

If C is chosen, then

$$R_1 = \frac{Q}{H_0 \omega_0 C}, \tag{84}$$

$$R_2 = \frac{2Q}{(1 - H_0)\,\omega_0 C}, \tag{85}$$

$$R_3 = \frac{(1 - H_0)\,Q}{\left(H_0^2 - H_0 + 2Q^2\right)\omega_0 C}. \tag{86}$$

3.4 EQUAL COMPONENTS

$$\begin{aligned} R_1 = R_2 = R_3 &= R \\ C_1 = C_2 &= C \end{aligned} \tag{87}$$

$$H_0 = \frac{K}{4 - K} \tag{88}$$

$$\omega_0 = \frac{\sqrt{2}}{RC} \tag{89}$$

$$Q = \frac{\sqrt{2}}{4 - K} \tag{90}$$

References

[1] S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, 4th ed. McGraw Hill, 2015, ISBN: 978-0-07-802816-8.

[2] Maxim Integrated, "Minimizing Component-Variation Sensitivity in Single Op Amp Filters," Application Note 738, Jul. 22, 2002.

[3] Texas Instruments, "OA-28 Low-Sensitivity, Bandpass Filter Design With Tuning Method," Application Note SNOA373C, Apr. 2013.